Why does it work?
first, how things balance using some nice round numbers:

ASSUME A 6 inch long rod, with 3 POUNDS on the right and 6 pounds on the left.

The balance point would be:

we can then show that
as $w 2=\left(\frac{b}{a}\right) * w 1$
proof $\left(\frac{2}{4}\right) * 6=.5 * 6=3$
We can see from this that the unit of weight and the unit of length can be anything and the formula will still work. It can be pounds and inches, kg and feet, carts and inches, carots and leagues...... the units all cancel themselves out. If anyone wants to add units to the formulas, they will see that.
so we also know
SPECIFIC GRAVITY $=\frac{W A}{W A-W W}$

WA IS WEIGHT IN AIR

WW IN WEIGHT IN WATER

Lets use
a = length in air
b = length in air
$x=a \operatorname{arm}$ in water
$y=b$ arm in water

When we put them into the equation for sg we get
air weight $=w 1 *\left(\frac{b}{a}\right)$
water weight $=w 1 *\left(\frac{y}{x}\right)$
put these into the sg equation and we get

$$
w 1 *\left(\frac{b}{a}\right)
$$

$\mathrm{sg}=$

$$
\left(w 1 * \frac{b}{a}\right)-\left(w 1 * \frac{y}{x}\right)
$$

move w1 out on the bottom

$$
w 1 *\left(\frac{b}{a}\right)
$$

$\mathrm{sg}=$

$$
w 1 *\left(\frac{b}{a}-\frac{y}{x}\right)
$$

factor out the w1
$s g=\left(\frac{w 1}{w 1}\right) *\left(\frac{\frac{b}{a}}{\frac{b}{a}-\frac{y}{x}}\right)$

$$
\frac{w 1}{w 1}=1
$$

so the weight 1 on the left really does not come onto play, only the ratio of balance points.
$s g=\frac{\frac{b}{a}}{\frac{b}{a}-\frac{y}{x}}$
to simplify more we invert and multiply

$$
s g=\frac{1}{\left(\frac{a}{b}\right) *\left(\frac{b}{a}-\frac{y}{x}\right)}
$$

multiply bottom out
$s g=\frac{1}{\left(\frac{a b}{b a}-\frac{a y}{b x}\right)}$
$\left(\frac{a b}{b a}\right)=1$
$s g=\frac{1}{1-\frac{a y}{b x}}$

